Sinusoidal Behavior Lab

Many phenomena are periodic. For instance, the information below gives the number of hours $y$ of daylight for Seattle, Washington, as a function of the number of days $x$ after March 21.

$y$ (Hours of daylight)

March 21 is the vernal equinox, the date on which night and day are approximately of equal length. From March 21 until June 21 the number of hours of daylight in Seattle increases to a maximum of about 16. From June 21 until December 21 the number of hours of daylight decreases, reaching 12 again around September 22 (the autumnal equinox) and a minimum of about 8.5 on December 21. Then the number of hours of daylight starts to increase again. Some intermediate days on the graph are values for May 6 ($x = 46, y = 15$), August 14 ($x = 146, y = 14.75$), October 25 ($x = 218, y = 10.25$), and February 28 ($x = 344, y = 11$).

1. Plot the points suggested in the above paragraph and then join them in a smooth curve.

2. Find an equation that will model the number of daylight hours in Seattle.

3. Once you have the equation, you can use it as a predictor for the number of daylight hours for any specified date of the year. Predict the number of daylight hours for July 30.
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4. Your equation would probably be more useful if \( x \) represented the day of the year beginning with January 1 instead of the day after March 21.

a. Rewrite your original equation so that \( x \) represents the day of the year beginning with January 1.

b. Graph your new equation on the same axes as the original equation.

c. Using the new equation, verify the number of daylight hours on July 30.

d. Solve your new equation for \( x \) and give its domain and range.

e. On what day(s) during the year would you expect 9 hours and 35 minutes of daylight?

Pictured at right is an oil well pump jack. As the motor on the jack turns, the walking beam rocks back and forth, pulling the pump rod in and out of the well. The distance of point \( P \) from the ground, \( d \), varies sinusoidally with time as the pump runs.

1. Suppose that the pump is started at time \( t = 0 \) seconds. One second later, \( P \) is at its highest, \( d = 14 \) feet above the ground. 2.5 seconds after that, \( P \) is at its next low point, \( d = 10 \) feet. Sketch the graph of this sinusoid in the space at right.

2. Write an equation modeling the distance, \( d \), with respect to the time, \( t \).

3. Graph your equation in the space at right and compare it to your sketch in 1).

4. Predict \( d \) when \( t = 9 \).
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5. For how long a time does $P$ stay above 11 feet each cycle?

6. Solve the equation for $t$ and give its domain and range.

7. At what time(s) during the first 10 seconds is $P$ 13 feet 2 inches above the ground?

8. The angle of elevation, $\theta$, which the walking beam makes with the ground varies with time. Find the maximum positive degree measure which $\theta$ attains.

9. When $\theta = \frac{\pi}{20}$, what is the value of $d$ and what is the first positive value of $t$ for this value of $d$?

Suppose the top of a ferris wheel 40 feet in diameter is 45 feet above the ground. It takes 10 seconds for you to reach the bottom after the last people have boarded. If the wheel turns at a uniform speed of 2 revolutions per minute, write a sine function that describes the distance $d$ feet you are above the ground at time $t$ seconds. The average length of the ride is 5 minutes.

1. Make a sketch (don't use an equation just yet) of the problem, including the ferris wheel and the sinusoidal curve describing your distance above the ground plotted against time.
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2. Determine the sine function that models the problem.

3. Graph the function in the space at right and compare it to the sketch you made in 1).

4. Predict the distance from the ground you would be after 80 seconds.

5. Predict the distance from the ground you would be after 5 minutes.

6. Solve your function for $t$ and state its domain and range.

7. At what time(s) during the first three revolutions of the ferris wheel are you 25 feet above the ground?

You are applying for a job with Y. O. Ming Mining Company. Since your work will involve using inverse circular functions, Mr. Ming poses some problems for you to solve during your interview to see how useful you would be to his company.

1. Find the exact value of each of the following:
   a. $\cos^{-1}\left(\frac{-\sqrt{2}}{2}\right)$
   b. $\cot^{-1}\left(-\sqrt{3}\right)$
   c. $\sec^{-1}(2)$
   d. $\sin^{-1}(2)$ (Your resume is shredded if you miss this one!)  
   e. $\tan(\tan^{-1}\left(\frac{2}{5}\right))$
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2. Sketch the graph.
   a. $y = \sin^{-1}\left(\frac{x}{2}\right)$
   b. $y = \sec^{-1}(x - 3)$
   c. $y = 2\tan^{-1}(x)$

Mr. Ming is satisfied with your performance and assigns you to the Uranium Mining Project (UMP). A layer of ore beneath the ground has surfaces that are sinusoidal in cross section as shown at the right. UMP plans to drill a vertical mine shaft through the ore layer and then dig a horizontal tunnel, again going through the ore layer. Your job is to find out how far this horizontal tunnel goes through the ore.

You set up a Cartesian coordinate system with the $x$-axis at ground level and $x$ and $y$ in meters. Your find that the top and bottom surfaces have the following equations:

Top: $y = -100 + 60\cos\left(\frac{\pi}{250}(x - 80)\right)$
Bottom: $y = -120 + 60\cos\left(\frac{\pi}{250}(x - 80)\right)$

3. Solve each equation for $x$ in terms of $y$.

4. How far will the horizontal shaft go through the formation if it is dug at a depth of
   a. $y = -90$ m?
   b. $y = -170$ m?